Reflectivity Calculations

1 Introduction

The radar reflectivity factor (Z) is defined as the sum of the drop diameter to the sixth power and is valid when the particle is smaller than about a tenth of the wavelength.

$$Z = \sum N D^6 \tag{1}$$

The purpose of the exercise is to get a "feel" for the magnitude of this parameter for different precipitation types - drizzle, rain - and to get use to the "units".

The idea is to do "back of the envelope" type computations given common characteristics of drizzle and rain.

2 Drizzle

Drizzle is a very light precipitation type. It falls relativelyt slowly and consists of small particles of about 100 microns or 0.1 mm in diameter.

Because drizzle rain rates are so small, it is difficult to estimate as a rain rate. But since it is a result of the condensation process, the liquid water content can be estimated. Typical liquid water contents (LWC) are of the order of 0.1-1.0 g/m^3. So, we assume that the LWC of drizzle is about the same.

The volume of a drizzle drop of size d is

$$V_d = \pi over 6 d^6 \tag{2}$$

The mass of a drizzle drop is

$$m_d = \rho_w V_d \tag{3}$$

whereρ_d is the density of water.

So in one cubic meter, the number of drizzle drops is

$$N_d = LWC/m_d \tag{4}$$

Therefore the radar reflectivity factor is

$$Z = N_d d^6 \tag{5}$$

3 Rain

Rain is usually reported in units of "mm/h". This is a volume and not a mass measurements.

A typical rain rate is 1 mm/h and a typical rain drop size is 1 mm. For derivation of the number of drops, assume that we have a 1 m^2 surface to catch the rain.

So in

$$R\ [mm/h]\ = LWC_v V_t \delta T \tag{6}$$

where LWC_v is the LWC expressed as volume of drops per unit volume of air. V_t is the terminal velocity of the particles and δT is 3600 s to make the units right.

Then

$$LWC_v = R/V_t/\delta T \tag{7}$$

and the number of drops is

$$N = LWC_v/v_r \tag{8}$$

where v_r is the volume of a single rain drop.

Then the rest is the same as above.

4 Z-R Relationships

The Marshall-Palmer relationship is the "classic" relationship between Z and R. In the absence of a validated regional Z-R relationship, this probably should be used as the default. It is the most defensible because it is the most quoted not because it is the most valid. There are many others.

$$Z = 200R^{1.6} \tag{9}$$

Note that Z is in units of $mm^6 m^{-3}$ and R is in units of mm/h in this equation.

Specific relationships have been developed for different weather regimes and situations but they remain to be proven. Richards and Crozier (1983) did a study for southern Ontario and found an "all-weather" relationship.

$$Z = 295R^{1.43} \tag{10}$$